To give an example it can be shown that for a vortex chamber investigated in [7] with R_K = 12.5 mm, r_c = 1.6 mm, h = 15 mm, k = 1.8 the quantity ξ_A = 0.187 is ~40 times smaller than the overall resistance coefficient ζ = 7.95 [2].

NOTATION

r, x, φ, coordinates; v, w; V, W, radial and circular velocity components for the boundary layer or the main flow, respectively; p, pressure; ρ, density; ν_T, turbulence analog of kinematic viscosity coefficient; δ, thickness of boundary layer.

LITERATURE CITED

1. V. P. Koval' and S. L. Mikhailov, Teploénergetika, No. 2, 25 (1972).
2. V. P. Koval' and S. L. Mikhailov, Teploénergetika, No. 5, 31 (1972).
3. M. I. Deveterikova and P. M. Mikhailov, Énergomashinostr., Tr. Leningr. Politekh. Inst., 310, 122 (1970).
4. V. A. Uspenskii and V. I. Solov'ev, Inzh.-Fiz. Zh., 18, No. 3, 459 (1970).
5. K. Borho, W. Gregor, and H. Reichert, Chem. Eng. Tech., 44, No. 16 (1972).
6. L. V. Kantorovich and V. I. Krylov, Approximate Methods of Higher Analysis [in Russian], Fizmatgiz, Moscow (1962).
7. D. T. Karpukhovich, Teploénergetika, No. 11, 79 (1960).

EFFECT OF RADIATION ON THE SUPERSONIC FLOW OF A VISCOUS IONIZED GAS PAST BLUNT BODIES

L. B. Gavin

UDC 533.6.011

The supersonic flow of a viscous monatomic ionized gas past blunt bodies is investigated. The effect of radiation on the field of flow and on the heat flux transmitted to the wall is shown.

When a gas flows past a blunt body at supersonic speed, the presence of the high temperatures that arise in the wake of a shock wave leads to changes in the physicochemical properties of the gas because there is excitation of the internal degrees of freedom of the molecules, dissociation, ionization, and radiation. Depending on whether the time taken by these processes is comparable to the characteristic time of flow in the shock layer or is much shorter, the conditions of flow past the body will be nonequilibrium or equilibrium conditions. In the first case we must consider the actual kinetics of the nonequilibrium processes.

The flow of a monatomic nonequilibrium-ionized radiating gas in a shock layer was considered in [1-4], but only in the ideal-gas model. In [5] the case of flow of a viscous nonequilibrium-ionized gas was analyzed without taking account of radiation.

In the present article we investigate the flow of argon past blunt bodies, with the following ionization reactions taking place in the gas:

$$A + M \rightleftarrows A^* + M, \quad A^* + M \rightleftarrows A^+ + M + e, \tag{1}$$

$$A + h\nu \rightleftarrows A^+ + e, \tag{2}$$

where A, A* denote the atom in the ground state and an excited state; A^+ denotes a singly charged ion; e denotes an electron; hν denotes a photon; M is A or e.

M. I. Kalinin Leningrad Polytechnic Institute.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 29, No. 4, pp. 699-705, October, 1975. Original article submitted December 25, 1974.

The rates of the collision reactions (1) involving an excited level are given in [3], and the rate of reaction (2), which describes ionization from the ground level, is given in [6].

The initial system of equations describing the flow of a viscous heat-conducting constant-temperature nonequilibrium-ionized radiating gas includes the equations of continuity, motion, energy, and state, and also the relaxation equation for the rate of ionization and the radiation-transfer equation.

In order to solve the problem, these equations are written in a coordinate system related to the body and are transformed in the framework of a known model for a thin shock layer [7]. Neglecting the change in pressure across the layer, taking account of the ambipolar character of the diffusion, and considering the radiation-transfer equation in the plane-layer approximation, we arrive at the following system of equations:

$$\frac{\partial}{\partial x}(r\rho u)+\frac{\partial}{\partial y}(r\rho v)=0, \tag{3}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{4}$$

$$\rho u\frac{\partial\alpha}{\partial x}+\rho v\frac{\partial\alpha}{\partial y}=\frac{\partial}{\partial y}\left(\rho D_A\frac{\partial\alpha}{\partial y}\right)+m_a(\dot{n}_{aa}+\dot{n}_{ea}+\dot{n}_R), \tag{5}$$

$$\rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial y}=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial y}\left(\rho D_A\frac{\partial h}{\partial\alpha}\frac{\partial\alpha}{\partial y}\right)+u\frac{dp}{dx}+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{dg_R}{dy}, \tag{6}$$

$$p=\rho RT(1+\alpha), \tag{7}$$

$$\cos\theta\frac{dI_\nu}{dy}=\rho(1-\alpha)\varkappa_\nu(S_\nu-I_\nu). \tag{8}$$

In (8) θ is the angle between the y axis and the direction of propagation of the photons, and S_ν is the source function,

$$S_\nu=\frac{\alpha^2}{1-\alpha}\,\frac{1-\alpha_E}{\alpha_E^2}\,B_\nu(T). \tag{9}$$

Under equilibrium conditions the degree of ionization is $\alpha = \alpha_E$, i.e., is given by the Saha equation, and the source function (9) reduces to the ordinary Planck function.

As boundary conditions on the shock wave (it is assumed that the shock wave is a surface of discontinuity of the gasdynamics parameters) we use the Rankine–Hugoniot relations, supplemented by the condition for the degree of ionization, $\alpha_S = \alpha_\infty$ for nonequilibrium flow and $\alpha_S = \alpha_E(T_S, p)$ for equilibrium flow. On the body u = 0, v = 0, T_W = const, $\alpha_W = \alpha_E(T_W, p)$.

In order to find the solution of the radiation-transfer equation on the basis of the plane-layer model, we must impose two conditions. The gas in front of the shock wave is slightly heated, and we therefore assume that it does not emit any radiation, i.e.,

$$I_{\nu s}^-=0, \tag{10}$$

and on the body we use the radiant-energy balance as a boundary condition:

$$I_{\nu w}^+=\delta B_\nu(T_w)+(1-\delta)I_{\nu w}^-, \tag{11}$$

where δ is the coefficient of blackness of the surface of the body.

As the first step in obtaining the solution in the entire subsonic region, we consider the flow in a neighborhood of a critical point. In this case we assume that all the dependent variables except the tangential component of velocity $u = u_1(y)x$, the quantity $r = x$, and the pressure determined by Newton's formula are functions of a single variable y [7]. Then the system (3)-(8) reduces to a system of ordinary differential equations.

We introduce the variable η and express u_1 and v in terms of $f(\eta)$:

$$\eta = \sqrt{\frac{V_\infty}{L\rho_s\mu_s}} \int_0^y \rho dy, \quad u_1 = \frac{1}{2} \frac{V_\infty}{L} f'(\eta), \quad v = -\frac{V_\infty}{L} f(\eta) \frac{dy}{d\eta}. \tag{12}$$

Now we change to dimensionless coordinates:

$$\bar{v} = \frac{v}{V_\infty}, \quad \bar{u}_1 = \frac{u_1 L}{V_\infty}, \quad \bar{\rho} = \frac{\rho}{\rho_\infty}, \quad \bar{p} = \frac{p}{\rho_\infty V_\infty^2}, \quad \bar{T} = \frac{RT}{V_\infty^2},$$
$$\bar{q}_R = \frac{q_R}{\rho_\infty V_\infty^3}, \quad \xi = \frac{\eta}{\eta_s}, \quad \varphi = \frac{df}{d\eta}, \quad \bar{\varphi}(\xi) = \int_0^\xi \varphi d\xi. \tag{13}$$

Finally, Eqs. (4)-(6) in a neighborhood of a critical point take the form (we have omitted the bar above the dimensionless quantities)

$$\frac{1}{\eta_s^2} \frac{d}{d\xi}\left(l \frac{d\varphi}{d\xi}\right) + \bar{\varphi} \frac{d\varphi}{d\xi} - \frac{1}{2}\varphi^2 + \frac{4}{\rho}(1-k) = 0, \tag{14}$$

$$\frac{1}{\eta_s^2} \frac{d}{d\xi}\left(\frac{l}{\mathrm{Sc}} \frac{d\alpha}{d\xi}\right) + \bar{\varphi} \frac{d\alpha}{d\xi} + \frac{m_a}{\rho}(\dot{n}_{aa} - \dot{n}_{ea} + \dot{n}_R) = 0, \tag{15}$$

$$\frac{1}{\eta_s^2} \frac{d}{d\xi}\left(\frac{l}{\mathrm{Pr}} \frac{dT}{d\xi}\right) + (1+\alpha)\bar{\varphi} \frac{dT}{d\xi} + \frac{1}{\eta_s^2} \frac{l}{\mathrm{Sc}} \frac{d\alpha}{d\xi} \frac{dT}{d\xi} -$$
$$-\left(T + \frac{2}{5}T_j\right) \frac{m_a}{\rho}(\dot{n}_{aa} + \dot{n}_{ea} - \dot{n}_R) - \frac{2}{5}\bar{\varphi}(1) \frac{dg_R}{d\xi} = 0 \tag{16}$$

with the following boundary conditions:

$$\xi = 0: \varphi = 0, \ \alpha = \alpha_w, \ T = T_w; \tag{17}$$

$$\xi = 1: \varphi = 2, \ \alpha = \alpha_s, \ T = T_s; \tag{18}$$

here

$$l = \rho\mu/\rho_s\mu_s, \quad \mathrm{Pr} = 5R\mu/2\lambda, \quad \mathrm{Sc} = \mu/\rho D_A. \tag{19}$$

In order to solve the system of equations, we must know l, Pr, Sc, i.e., the transfer coefficients, as functions of the thermodynamic parameters of the gas. In most calculations the transfer coefficients were determined by the simplest classical theory [8].

In [5] μ and λ were first calculated to high degrees of approximation, according to [9, 10], for the case of nonequilibrium flow, and estimates were given for this correction to the field of the flow and the convective flux transmitted to the wall for the case of flow past blunt bodies disregarding radiation. In the present article we investigate the effect that taking account of the transfer coefficients in higher approximations produces both on the convective and on the radiant heat flux.

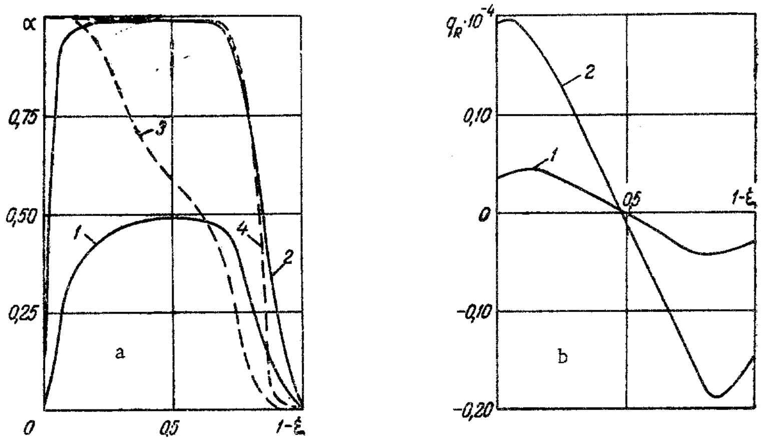

Fig. 1. Profiles of the degree of ionization (a) for nonequilibrium (1a, 2a) and equilibrium (3a, 4a) conditions and the radiant heat flux (b) in the shock layer for $M_\infty = 20$ (1a, 3a, 1b). $M_\infty = 22$ (2a, 4a, 2b); $p_\infty = 50$ n/m^2, $T_\infty = 300$°K, $\alpha_\infty = 10^{-3}$, L = 0.06 m, T_W = 2000°K (α, ξ are dimensionless quantities, q_R has the dimensions of kW/m^2).

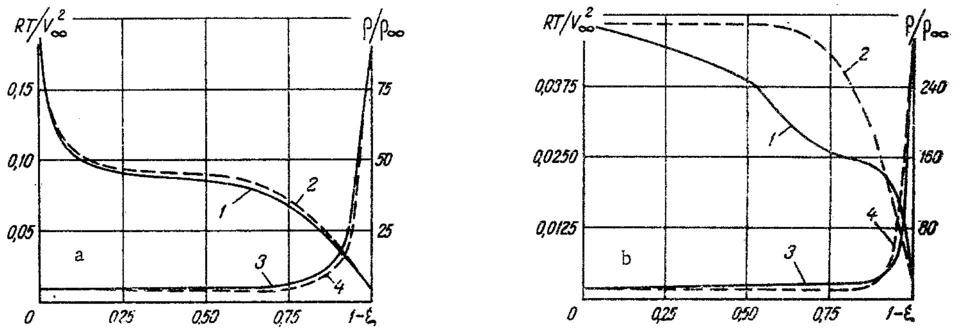

Fig. 2. Profiles of temperature (1, 2) and density (3, 4) in the shock wave, with radiation taken into account (1, 3) or disregarded (2, 4): a) for $M_\infty = 22$, $p_\infty = 50$ n/m^2, $T_\infty = 300$°K, $\alpha_\infty = 10^{-3}$, L = 0.06 m, T_W = 2000°K; b) for $M_\infty = 38$, $p_\infty = 100$ n/m^2, T_∞ = 300°K, $\alpha_\infty = 10^{-3}$, L = 0.2 m, T_W = 2000°K (RT/V_∞^2, ρ/ρ_∞, ξ are dimensionless quantities).

The expression for the spectral flux of radiant energy, obtained in the plane-layer approximation for the case $\delta = 1$, neglecting radiation from the surface of the body because of its low temperature, has the form

$$q_{R\nu}(\tau_\nu) = 2\pi \int_0^{\tau_\nu} S_\nu E_2(\tau_\nu - t_\nu)\, dt_\nu - 2\pi \int_{\tau_\nu}^{\tau_{\nu s}} S_\nu E_2(t_\nu - \tau_\nu)\, dt_\nu, \qquad (20)$$

$$E_2(z) = \int_1^\infty w^{-2} \exp(-wz)\, dw, \quad \tau_\nu = \int_0^y \rho(1-\alpha)\varkappa_\nu dy, \quad \varkappa_\nu = \sigma_j/m_a. \qquad (21)$$

In calculating the integrated flux $q_R = \int_{\nu_j}^{\infty} q_{R\nu} d\nu$ we make a step-function approximation to the coefficient of absorption with respect to the frequency, making use of the experimental value of the cross section of photoionization from the ground state, $\sigma_j = 34 \cdot 10^{-18}$ cm^2, according to the data of [11].

Some results of the calculations are shown in Figs. 1, 2a (nonequilibrium conditions) and Figs. 2b and 3 (equilibrium conditions). The solution was found by the factorization method, using two iterations, on the BÉSM-4 electronic digital computer

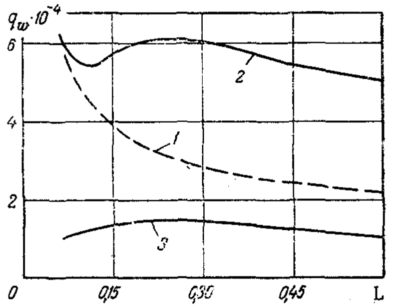

Fig. 3. Heat flux transmitted to the wall as a function of the blunting radius: 1) convective, disregarding radiation, 2) convective, with radiation taken into account, 3) radiant; $M_\infty = 38$, $p_\infty = 100$ n/m², $T_\infty = 300$°K, $\alpha_\infty = 10^{-3}$, $T_W = 2000$°K (q_W, kW/m², L, m).

TABLE 1. Radiant Heat Flux Transmitted to the Wall, q_{Rw} in kW/m², and Maximum Value of q_R in the Shock Layer, q_{Rm} in kW/m²

Method of calculation of the tranfer coefficients	q_{Rw}	q_{Rm}	Method of calculation of the tranfer coefficients	q_{Rw}	q_{Rm}
$l=1$, Pr=2/3, Sc=1	109,6	132,9	μ_1, λ_2 [9, 10]	108,0	143,9
According to [8]	112,4	147,1	μ_2, λ_4 [9, 10]	113,4	148,3

Internal iterations were used for determining the energy from the shock wave, and external iterations were used for correcting the radiation terms.

Figure 1 shows the distribution of $\alpha(\xi)$, $\alpha_E(\xi)$, and $q_R(\xi)$. It can be seen that as M_∞ increases, the relaxation region contracts to the wave and the flow approaches equilibrium flow and that when there is motion across the shock layer, the radiant flux changes sign, carrying away energy through the shock wave upward along the flow and to the body. There is partial screening of the radiant flux by the region near the wall. As M_∞ increases, the minimum and maximum values of q_R increase in value, and so does q_{Rw}. The quantity q_{Rw} represents an increasingly large part of the corresponding convective flux. For example, for $M_\infty = 20$ and 22 the convective and radiant fluxes transmitted to the wall are equal, respectively, to 6372 and 307 and 13,750 and 1432 kW/m².

Figure 2a shows the profiles of the dimensionless temperature and density in the shock layer for gas flow taking account of radiation and disregarding radiation. There is a sharp drop in temperature near the shock wave because the rate of the reactions in this region is very high (Fig. 1a), causing large losses in the energy of the gas. It can be seen that when radiation is taken into account, there is a decrease in temperature and a corresponding increase in density in a large part of the shock layer; this is due to the cooling of the gas as a result of the removal of radiant energy.

Figure 2b shows the profiles of the dimensionless temperature and density in the shock layer for conditions corresponding to equilibrium flow past the body. In this case the shock wave becomes optically denser than in the nonequilibrium case. While in most of the shock layer the radiant cooling leads to a drop in temperature, the temperature in the region near the wall increases because the radiant energy is absorbed more intensively than in the nonequilibrium case.

Figure 3 shows how the convective and radiant heat fluxes transmitted to the wall vary with the blunting radius. If radiation is disregarded, the convective flux decreases as L increases. If the radiation is taken into account, there is a substantial increase in the convective flux because the temperature near the wall rises, and so the behavior of the flux as L increases is nonmonotonic. It can be seen that under these conditions the radiant flux amounts to a substantial fraction of the corresponding convective flux.

In our study we also made an estimate of the effect produced on the radiant flux q_R by transfer coefficients calculated to higher approximations. In calculating μ_1, μ_2, λ_2, λ_4, we used the interaction potentials for partially ionized argon according to [9, 10]. As an example, we show in Table 1 the results obtained by calculating q_{Rw} and q_{Rm} for $M_\infty = 18$, $p_\infty = 100$ n/m², $T_\infty = 300$°K, $\alpha_\infty = 10^{-3}$, $L = 0.04$ m, $T_W = 2000$°K.

The radiant heat flux is an integral function of the distribution of the gas-dynamics parameters in the shock layer, and therefore, as was to be expected, there is smoothing of q_R for different methods of calculating the transfer coefficients, resulting in a slight difference in q_{Rw} and q_{Rm}.

NOTATION

A, A*, atom in ground state and excited state; A^+, singly charged ion; x, y, coordinates; u, v, components of velocity along the x and y axes, respectively; r, distance from the axis of symmetry of the body; ξ, dimensionless coordinate across the shock layer; V_∞, M_∞, gas velocity and Mach number in the incoming flow, respectively; p, ρ, T, α, pressure, density, temperature, degree of ionization of the gas, respectively; α_E, equilibrium degree of ionization; $h = (5/2)RT(1 + \alpha) + \alpha RT_j$, specific enthalpy of the mixture; R, specific gas constant; ν_j, T_j, frequency and temperature of ionization; $B_\nu(T)$, Planck function; $\varkappa_\nu$, coefficient of absorption of a unit mass of atomic gas; τ_ν, optical coordinate; σ_j, cross section of photoionization from the ground state; I_ν^+, I_ν^-, spectral intensity of radiation propagated in the positive (+) and negative (–) directions of the ξ axis; $q_{R\nu}$, q_R, spectral and integrated fluxes of radiant energy; q_{Rm}, maximum value of radiant flux in the shock layer; m_a, m_e, masses of atom and electron; $\dot{n}_{aa}$, $\dot{n}_{ea}$, rates of ionization by atomic and electron–atomic collision; $\dot{n}_R$, rate of photoionization; k, ratio of densities before and immediately after the shock wave; μ, λ, D_A, coefficients of viscosity, thermal conductivity, and ambipolar diffusion, respectively; μ_1, μ_2, coefficients of viscosity in the first and second approximations; λ_2, λ_4, coefficients of thermal conductivity in the second and fourth approximations; Pr, Prandtls number; Sc, Schmidt number; $l = \rho\mu/\rho_s\mu_s$, dimensionless parameter. Indices: ∞, s, w, gas parameters in the incoming flow, immediately after the shock wave, and on the body, respectively.

LITERATURE CITED

1. M. D. Kremenetskii, N. V. Leont'eva, and Yu. P. Lun'kin, Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 121 (1971).
2. A. K. Kondrya and N. V. Leont'eva, Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 179 (1972).
3. N. V. Leont'eva, Yu. P. Lun'kin, and A. A. Fursenko, Inzh.-Fiz. Zh., 25, 681 (1973).
4. N. V. Leont'eva, Yu. P. Lun'kin, and A. A. Fursenko, Inzh.-Fiz. Zh., 27, 421 (1974).
5. L. B. Gavin and Yu. P. Lun'kin, Inzh.-Fiz. Zh., 26, 424 (1974).
6. J. H. Clarke and C. Ferrari, Phys. Fluids, 8, 2121 (1965).
7. L. A. Ladnova, Vestn. Leningr. Gos. Univ., No. 2, 91 (1969).
8. M. Y. Jaffrin, Phys. Fluids, 8, 606 (1965).
9. R. S. Devoto, Phys. Fluids, 9, 1230 (1966).
10. R. S. Devoto, Phys. Fluids, 10, 354 (1967).
11. J. A. Samson, J. Opt. Soc. Amer., 54, 420 (1964).